# Online Appendix for 'Birds of a Feather ... Enforce Social Norms? Interactions among Culture, Norms, and Strategy'

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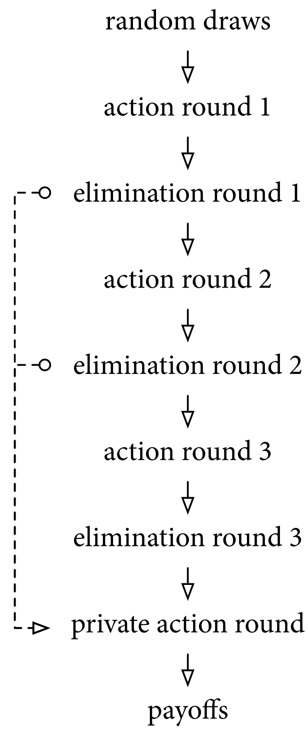
This online appendix discusses details of the formal model in Section 6 of the paper. It aims to provide a self-contained description of the formal analysis. This means that the following discussion overlaps substantially with that of Section 6 – but the discussion also complements the sometimes-informal exposition of Section 6 by filling in some details of the equilibrium and the payoff functions, making some features (such as the model timeline) more explicit, and providing details of the proof of Proposition 1.

The paper's formal model is described in Appendix A. The 'standard norm equilibrium' is specified in Appendix B. Proposition 1 is restated, a proof is outlined, and examples of the incentive constraints induced by the 'standard norm equilibrium' are provided in Appendix C.

## A Model and Timeline

There is an organization with $N = 5$ positions to be filled. The employee in position $n$ will choose a public course of action $a_n \in \{L, R\}$ and a private action $b_n \in \{L, R\}$. (Hereafter, we simply call $a_n$ an *action*; $b_n$, a *private action*.) Employees are drawn randomly from an infinite pool with two *kinds* of employees, in equal proportion: those who prefer action $L$ and those who prefer action $R$. Let $\tau_n \in \{L, R\}$ denote the kind of employee in position $n$. Amongst each kind, proportion $1 - \gamma$ are 'regular' types, labeled $l$ and $r$, whereas proportion $\gamma$ are 'strong' types, labeled $L$ and $R$. The relative proportions of the four employee types $(L, l, r, R)$ are thus $\left(\frac{\gamma}{2}, \frac{1-\gamma}{2}, \frac{1-\gamma}{2}, \frac{\gamma}{2}\right)$. Our analysis will focus on the case of vanishingly rare strong types, $\gamma \to 0$.

The game consists of two parts. In Part I, after employees are randomly drawn, up to three rounds are played; each round consists of an action round followed by an elimination round. In each action round of Part I, every employee $n$ simultaneously chooses (or re-chooses, in the second and third action rounds) an action $a_n \in \{L, R\}$. In each elimination round of Part I, each employee simultaneously indicates which other employees to vote against; any employee receiving a majority ($> N/2$) of 'against' votes is immediately eliminated and replaced with a new random draw from the pool. Each employee who is not eliminated in a given round incurs an 'enforcement' cost of $\epsilon > 0$ for each employee who is eliminated in that round. At the end of the third elimination round, the game proceeds to Part II. Alternatively, after any elimination round, if nobody voted against anybody else in that round, then the game proceeds immediately to Part II. In Part II, every employee $n$ simultaneously chooses a private action $b_n$. All employees' action and voting choices are public. To sum up, the timeline of the game is:




The dotted-line arrows highlight – as mentioned above – that at the end of a given elimination round, if nobody voted against anyone else in that round, then the game skips ahead to the private action round.

For the second and third action rounds of Part I, the action choice $a_n$ for each position replaces the corresponding action choice from the previous action round; so, only the last-played action round of Part I counts. If an employee $n$ is eliminated (and replaced) in the third elimination round, his action choice $a_n$ from the third action round is retained; his replacement does not get to re-choose $a_n$, but gets to choose $b_n$ in the private action round of Part II.

At the end of the game, final payoffs are realized. The regular employee in position $n$ at the end of the game receives utility

$$U_n = U + (VJ_{a_n=\tau_n} + vJ_{b_n=\tau_n}) + \sum_m (WJ_{a_m=\tau_n} + wJ_{b_m=\tau_n}) - k_n\epsilon$$

where $J_X = 1$ if condition $X$ is satisfied and equals $-1$ if it is not; and where $k_n$ is the number of other employees that were eliminated in rounds where employee $n$ was present and survived. Here, $U > 0$ is some benefit from being part of the organization, $V, v > 0$ are the benefits/costs from the employee's own choices, and $W, w > 0$ are benefits/costs that an employee enjoys/suffers from the choices of others in the organization. For a regular employee $i$ who is eliminated before the game ends, her final payoff is $-k_i\epsilon$, where $k_i$ is the number of other employees that were eliminated in rounds where $i$ was present and survived.

Strong-type employees (types $L$ and $R$) choose mechanically. We specify their strategies in Appendix B. Essentially, they always take their preferred action, and vote against employees who they believe are sufficiently likely to be of the opposite kind.

# B The 'Standard Norm Equilibrium'

To characterize the 'standard norm equilibrium', we start with some terminology, turn to describe the strategies, then specify the belief-updating rule.

**Terminology** Here, we reproduce – from Section 6 of the paper – the notation for perceived beliefs (e.g., $l/r/R$), the concepts of $L$- and $R$-norms and $L$- and $R$-majorities; and associated notation. The reader who is already familiar with Section 6 may skip ahead to the 'Strategies' heading.

Recall that a new random draw from the pool is a mixture of types $L/l/r/R$ with 'prior' probabilities $\frac{\gamma}{2}/\frac{1-\gamma}{2}/\frac{1-\gamma}{2}/\frac{\gamma}{2}$. At any decision node, we call public beliefs about an employee's type the *perceived type* of that employee. At any information set, a perceived type always corresponds to some nonempty subset of the four types with probabilities in relative proportion to the prior (e.g., $l/r$ in relative proportions $\frac{1}{2}/\frac{1}{2}$).

There are only three kinds of perceived types that occur in the 'standard norm equilibrium', on- or off-path. For perceived types $l$, $L$, and $l/L$, i.e., the employee is of type $l$ or $L$ with probability one, we say the employee is *$L$-revealed.* Analogously, for perceived types $r$, $R$, and $r/R$, we say the employee is *$R$-revealed.* Finally, for perceived types that include both $l$ and $r$, say $L/l/r$ or $l/r$, we say the employee is *unrevealed.* Notice that for small $\gamma$, unrevealed employees are roughly equally likely to be either type-$l$ or -$r$.

We say that an employee *acts like kind $L$* if she chooses course of action $L$. She *votes like kind $L$* if she votes against, and only against, $R$-revealed employees. (Acting and voting like type $R$ is analogous.) A type-$l$ or $L$ employee *acts and votes with her kind* if she acts and votes like kind $L$; analogously for a type-$r$ or -$R$ employee. An *$L$-norm* (and analogously, an *$R$-norm*) means that all regular employees who are $L$-revealed or unrevealed, act and vote like type $L$.

At any decision point, let $N_L$ and $N_R$ be the numbers of $L$-revealed and $R$-revealed employees. We say that there is an *$L$-majority* if $N_L > N_R$; an *$R$-majority* if $N_R > N_L$; and *no majority* if $N_R = N_L$.

**Strategies** Strong types $L$ and $R$ simply follow a mechanical rule: they act and vote according to their type in every round, regardless of their perceived type. Also, in the third (and final) elimination round and in choosing a private action, each regular employee always votes and acts according to her own kind.

It remains to specify strategies for regular employees for decision points up until (and excluding) the third elimination round. At each decision point, each employee's choice will condition on her type, her kind of perceived type ($L$-revealed, $R$-revealed, or unrevealed), and the kinds of perceived types of all current employees. We will distinguish between correctly-perceived revealed employees whose perceived type matches their kind (e.g., a type-$r$ who is $R$-revealed) and wrongly-perceived revealed employees (e.g., a type-$r$ who is $L$-revealed). Note that employees are never wrongly-perceived on the equilibrium path.

If there is no majority at the start of an action or elimination round, then each employee acts or votes with her own kind in that round – with the following exception: in an elimination round, a revealed but wrongly-perceived employee will vote against nobody.

If there is an $L$-majority (the case of an $R$-majority is analogous), then all correctly-perceived type-$l$'s and all unrevealed employees follow the $L$-norm, while all $R$-revealed employees (who may be correctly- or wrongly-perceived) act and vote as their type. Finally, consider wrongly-perceived type-$r$ employees. In action rounds, if there is a $L$-majority of at least two ($N_L - N_R \geq 2$), then they follow the $L$-norm; with a $L$-majority of one, they act with their kind. In each elimination round, wrongly perceived type-$r$'s follow the $L$-norm – except in the case $N_L = 3, N_R = 2$, where they vote with their kind.

**Beliefs** We describe how beliefs are updated in action and elimination rounds, excluding the last elimination round. (Given that employees cannot be eliminated by others after the last elimination round, belief updating at that point has no payoff consequences.)

Consider action rounds that start with no majority of either kind. Unrevealed employees who take action $L$ have their perceived type updated by removing $r/R$ from their set of possible types (e.g., from $L/l/r$ to $L/l$); so these employees become $L$-revealed. employees who take action $R$ are updated symmetrically. Revealed employees are not updated regardless of their actions.

Consider action rounds that start with an $L$-majority (the case of an $R$-majority is analogous). An unrevealed employee who follows the $L$-norm has her perceived type updated by removing $R$ from her set of possible types (e.g., from $l/r/R$ to $l/r$); so she remains unrevealed. $R$-revealed employees do not have their perceived type updated regardless of their action. Finally, an unrevealed or $L$-revealed employee who violates the $L$-norm (by taking action $R$) becomes $R$-revealed. Specifically: if her perceived type included $R$, then she becomes perceived to be $R$; otherwise, she becomes perceived to be $r$.

Now, turn to elimination rounds (except the last round). Consider elimination rounds that start with no majority. Unrevealed employees who vote as kind-$L$ have their perceived type updated by removing $r/R$ from their set of possible types; so these employees become $R$-revealed. Unrevealed employees who vote as type-$R$ are updated symmetrically. Revealed employees are not updated regardless of their actions. An unrevealed employee who votes off-path (i.e., neither as kind-$L$ or -$R$) is perceived to be $l$; so she becomes $L$-revealed.[1] A revealed employee's perceived type is never updated regardless of how she votes.

Consider elimination rounds that start with an $L$-majority (the case of an $R$-majority is analogous). The updating rule here is analogous to that of action rules that start with a majority. $R$-revealed employees are not updated regardless of their vote. An unrevealed employee who follows the $L$-norm has her perceived type updated by removing $R$ from her set of possible types; so she remains unrevealed. $L$-revealed employees who follow the $L$-norm are not updated. Finally, an unrevealed or $L$-revealed employee who deviates in any way from following the $L$-norm becomes $R$-revealed. Specifically, if her perceived type included $R$ and she deviated by voting like kind $R$, then she becomes perceived to be $R$; otherwise, she becomes perceived to be $r$.

# C Equilibrium Existence and Constraints

We now restate Proposition 1 of the paper, making explicit that the result holds for $\gamma \to 0$. Our solution concept is perfect Bayesian equilibrium. Let the allowed parameter space (excluding $\gamma$) be

$$\Omega = \{(V, v, W, w, \epsilon) : V > 0, v > 0, W > 0, w > 0, \epsilon > 0\}$$

and let $\omega$ be some generic point in $\Omega$.

[1] Notice that the updating rule in this case is asymmetric; we could just have well have specified that an unrevealed employee who votes off-path becomes $R$-revealed. This assumption is for simplicity of analysis. Specifying a symmetric rule, such as that the employee is randomly revealed to be one kind or another, would not change the rest of the analysis.

**Proposition 1a** *There exists a closed subset $S$ of $\Omega$ with non-empty interior such that:*

1. *For any element $\omega$ in the interior of $S$, the 'standard norm equilibrium' exists for all $0 < \gamma < \underline{\gamma}$, for some $\underline{\gamma} > 0$.*
2. *If $\omega = (U, V, v, W, w, \epsilon) \in S$ then a) $w \geq \epsilon$ and b) if $\omega' = (U', V', v', W', w', \epsilon)$ is such that $U' > U$, $V' < V$, $v' > v$, $W' < W$, $\epsilon < w' < w$ then $\omega'$ is also in $S$.*
3. *For any $\omega \in \Omega$ but outside of $S$, the 'standard norm equilibrium' does* not *exist for all $0 < \gamma < \underline{\gamma}$, for some $\underline{\gamma} > 0$.*

The following observation helps to establish the Proposition, and also provides some insight into the comparative statics results.

**Observation 1** *Every incentive constraint that deters a one-step deviation of the 'standard norm equilibrium' takes one of the following forms:*

$$w \geq \epsilon; \tag{1}$$

*or*

$$U + v \geq \alpha_V V + \alpha_W W + \alpha_w w + \alpha_\epsilon \epsilon + O(\omega; \gamma) \tag{2}$$
$$\text{where}$$
$$\alpha_V > 0,\ \alpha_W > 0,\ \alpha_w > 0,$$

*and where $O(\omega; \gamma)$ is a term that varies across constraints and for which $\lim_{\gamma \to 0} O(\omega; \gamma) = 0$; or*

$$\alpha_U U + \alpha_v v + \alpha_V V + \alpha_W W + \alpha_w w + \alpha_\epsilon \epsilon + O(\omega; \gamma) \geq 0 \tag{3}$$
$$\text{where}$$
$$\alpha_U \geq 0,\ \alpha_v \geq 0,\ \alpha_V \geq 0,\ \alpha_W \geq 0,\ \alpha_w > 0,\ \alpha_w + \alpha_\epsilon > 0,\ \lim_{\gamma \to 0} O(\omega; \gamma) = 0,$$
$$\text{and } \alpha_U + \alpha_v + \alpha_V + \alpha_W > 0;$$

*or*

$$U + v \geq \alpha'_V V + \alpha'_W W + \alpha'_w w + \alpha'_\epsilon \epsilon + O(\omega; \gamma) \tag{4}$$
$$\text{where}$$
$$\alpha'_V \leq \alpha_V,\ \alpha'_W \leq \alpha_W,\ \alpha'_w \leq \alpha_w,\ \text{with at least one strict inequality,}$$
$$\text{and } \lim_{\gamma \to 0} O(\omega; \gamma) = 0;$$

*where $\alpha_V$, $\alpha_W$, $\alpha_w$ are the coefficients for an incentive constraint of the form (2) that holds in the 'standard norm equilibrium'.*

*There are a finite number of such incentive constraints, and there is at least one incentive constraint of each of the forms (1) and (2).*

Constraints of the form (1) ensure that regular employees are willing to incur the cost of eliminating opposite-kind employees in the third (final) elimination round. They capture the comparative-static result that the equilibrium exists only for high-enough $w$.

Constraints of the form (2) ensure, essentially, that in situations with a majority, the majority (action or voting) norm is followed by unrevealed regular employees of the opposite kind. Notice that the remaining comparative statics results of Proposition 1a are encapsulated in (2): such incentive constraints are satisfied for high $U$, high $v$, low $V$, low $W$, and low $w$. To understand these comparative statics, consider the tradeoffs involved for an unrevealed type in violating the majority-kind's norm. Doing so increases the chances of being eliminated, and thus is unattractive if the employee highly values survival ($U$) and the chance to take a private action ($v$). On the other hand, survival under a opposite-kind norm entails being hurt, on average, by others' actions (because everyone else conforms to the 'undesirable' norm) and by others' private actions (because an opposite-kind norm means that there are more employees of the opposite kind, on average, than of the same kind). Thus as an employee's dependence on others' actions ($W$ and $w$) increases, the attractiveness of following the norm and avoiding elimination diminishes. This effect is augmented by the following amplification mechanism: by violating the norm and revealing her kind, an employee may trigger a 'showdown' between the two kinds where everyone reveals their kind with their actions and votes. Such a showdown ends with the focal employee surviving in and only in those states where there is a majority of her own kind – that is, in those states where she benefits on net from the others' actions and private actions. As $W$ and $w$ increase, these potential 'showdown' benefits also increase, and her temptation to violate the norm and trigger such a showdown increases as well.

Constraints of the form (3) and (4) are trivially satisfied as $\gamma \to 0$. They correspond, for instance, to incentive constraints for regular employees of the majority kind not to violate the norm (and thus become perceived to be minority-type employees).

The 'standard norm equilibrium' exists if and only if all of these incentive constraints are satisfied. For now, let's assume that Observation 1 holds – more on this in Section C.1 – in which case we may proceed to prove Proposition 1a.

**Proof of Proposition 1a:** Note that the following argument assumes Observation 1, and thus is not a self-contained proof of Proposition 1a.

Consider any incentive constraint of the form (3). Suppose (1) holds; then

$$\alpha_U U + \alpha_v v + \alpha_V V + \alpha_W W + \alpha_w w + \alpha_\epsilon \epsilon > 0,$$

so any incentive constraint of the form (3) is satisfied for sufficiently small $\gamma$.

Now, consider any incentive constraint of the form (2). Consider any $\omega \in \Omega$ that satisfies the following auxiliary constraint

$$U + v > \alpha_V V + \alpha_W W + \alpha_w w + \alpha_\epsilon \epsilon. \tag{5}$$

For sufficiently small $\gamma$ and thus $O(\omega;\gamma)$, $\omega$ must also satisfy (2). It follows that if $\omega$ satisfies all auxiliary constraints of the form (5), then there exists $\underline{\gamma} > 0$ such that for all $\gamma \in (0, \underline{\gamma})$, $\omega$ satisfies all incentive constraints of the form (2). Analogously, we may show that if $\omega \in \Omega$ violates any constraint of the form

$$U + v \geq \alpha_V V + \alpha_W W + \alpha_w w + \alpha_\epsilon \epsilon, \tag{6}$$

then it violates the corresponding incentive constraint (2) for all sufficiently small $\gamma$.

Let $S'$ be the intersection of all half-spaces of the auxiliary-constraint form (5) and the half-space corresponding to the strict version of constraint (1); that is, the set of elements of $\Omega$ that satisfy all such auxiliary constraints plus the constraint $w > \epsilon$. Being the intersection of a finite number of open sets, it is itself open. Being the intersection of convex sets, it is itself convex. Note that it is nonempty; for instance, pick any $V, v, W, w, \epsilon$ with $w > \epsilon$, and pick sufficiently large $U$. Further, notice that each auxiliary constraint continues to hold if we increase $U$, increase $v$, decrease $V$, decrease $W$, or decrease $w$.

Let $S$ be the closure of $S'$, so that $S'$ is the interior of $S$. By our construction, it is immediate that $S$ satisfies part 2 of Proposition 1a. Consider any $\omega \in S'$. Note that for sufficiently small $\gamma$, such $\omega$ satisfies all auxiliary constraints of the form (5), thus satisfying all incentive constraints of the form (2) given. Further, $\omega$ (strictly) satisfies the incentive constraint (1). Each relevant constraint of the form (3) is satisfied because, as we have already established, the (stricter, for sufficently small $\gamma$) corresponding relevant constraint of the form (2) is satisfied for sufficiently small $\gamma$. Finally, as discussed above, for sufficiently small $\gamma$, all constraints of the form (3) are automatically satisfied. Thus the 'standard norm equilibrium' exists under $\omega$; it follows that part 1 of Proposition 1a holds.

Consider any point $\omega \in \Omega$ outside $S$. By our construction of $S'$ and thus $S$, either (i) $w < \epsilon$ or (ii) some constraint of the form (6) is violated at $\omega$. If (i) holds, then the 'standard norm equilibrium' does not exist under $\omega$ for any $\gamma$. If (ii) holds, then some corresponding incentive constraint (2) is violated, and the 'standard norm equilibrium' does not exist, for all sufficiently small $\gamma$. It follows that part 3 of Proposition 1a holds. ■

## C.1 Incentive Constraints

To verify Observation 1, we separately derived the incentive constraints associated with each one-step deviation from the 'standard norm equilibrium' for regular employees, and checked that each constraint took one of the forms (1)–(4). These derivations were performed in a brute-force fashion and do not generate much additional insight. Below, instead of reproducing all our calculations for every constraint, we show the calculations for a representative subset of constraints. (The rest of the calculations are available from the authors.) Each subsection discusses a particular class of possible one-step deviations and provides one or two examples.

### Final (third) elimination round, and private actions

We start by checking incentive-compatibility conditions for regular employees to vote their type in the final elimination round. Specifically, we claim that incentive-compatility holds here if and only (1) is satisfied.

Consider a type-$r$ employee $n$'s voting choice vis-à-vis another employee $m$. This choice impacts $n$'s payoff if and only if (i) $n$ is not eliminated, and (ii) $n$'s vote on $m$ is pivotal. In that case, by eliminating $m$, $n$ incurs a cost of $\epsilon$ and ensures $E\left[J_{b_n=\tau_m}\right] = 0$. If $m$ is $R$-revealed, then he will choose private action $R$ with probability 1; so, absent elimination, $E\left[J_{b_n=\tau_m}\right] = 1$. Similarly, if $m$ if $L$-revealed, then absent elimination, $E\left[J_{b_n=\tau_m}\right] = -1$. Thus, voting to eliminate an $R$-revealed $m$ changes $n$'s expected payoff by $-p(w+\epsilon)$ where $p$ is the probability that $n$ survives the vote and that $n$'s vote to eliminate $m$ is pivotal; voting to eliminate an $L$-revealed $m$ changes $n$'s expected payoff by $p(w-\epsilon)$. It follows that it is never optimal for $n$ to vote against an $R$-revealed $m$; it is optimal for $n$ to vote against an $L$-revealed $m$ if and only if $w \geq \epsilon$. Finally, note that any vote by $n$ against an unrevealed $m$ is never pivotal. To sum up, it is optimal for $n$ to vote as type $R$, as the equilibrium prescribes, if and only if $w \geq \epsilon$; i.e., if (1) holds. An identical argument holds if employee $n$ is type-$l$.

It is obviously optimal for each employee to act with her kind in the private action round, which is the last decision point in the game.

For the rest of this Appendix, we focus on rounds up till (and including) the third action round.

**Unrevealed employees (to follow the norms)**

Suppose there is a majority: say, $N_R > N_L$. In this case, the equilibrium specifies that all regular unrevealed employees will follow the $R$-norm. We have to separately consider the incentive constraints for (i) a regular $l$-type (minority) and (ii) a regular $r$-type (majority) to do so.

*Example (i)*: if the focal employee is of the minority type, these incentive constraints take the form of (2). Focus on (for example) the case of an unrevealed $l$-type where, at the start of the second action round, there is an $R$-majority with $N_R = 2$ and $N_L = 1$. We study whether it is incentive-compatible for the focal employee to follow the $R$-norm in the second action round. In the continuation equilibrium, the focal employee follows the $R$-norm, never reveals herself and thus is never eliminated.

At this point, note that the continuation equilibrium plays out differently, depending on whether any of the other unrevealed employees (including future replacements drawn from the pool) are strong $L$-types. For a given decision point, let's label an outcome as *atypical* if at least one employee who is present in at least one round of the continuation equilibrium is a strong type; otherwise, we label the outcome as *typical*. We call the first type of outcome 'atypical' because such outcomes are rare – strong-type employees are rarely drawn (when $\gamma$ is small). Also, note that in typical circumstances, unrevealed employees (being regular) never violate the norm and never subsequently get eliminated. In our example: conditional on a typical outcome, the initially-$L$-revealed employee will be eliminated and replaced in the second elimination round, everyone will follow the $R$-norm in the third action round, and the initially-$R$-revealed employees are eliminated with probability 1/4 in the final round. (Why 1/4? This outcome is realized if and only if both of the non-focal unrevealed employees, including the new replacement, are type-$l$ – which occurs with probability 1/4. In this case, they and the focal employee all vote to eliminate the initially-$R$-revealed employees.) Thus, conditional on a typical outcome, the expected payoff to the focal employee is (after some computation)

$$U + v - V - \frac{3w}{2} - 4W - \frac{3\epsilon}{2}. \tag{7}$$

What about atypical outcomes? Instead of explicitly tracing out the continuation equilibrium, we bound (coarsely) the focal employee's expected payoffs by considering the lowest and highest possible payoffs any employee can obtain in any history:

$$\begin{gathered} U_n \in [U_{\min}, U_{\max}] \quad \text{where} \\ U_{\min} = \min\{0, U - (V + v) - 4(W + w)\} - 12\epsilon \quad \text{and} \\ U_{\max} = U + (V + v) + 4(W + w). \end{gathered}$$

In addition, we can bound the probability of an atypical outcome. Any unrevealed employee is a strong type with probability $\leq \gamma$, and $n$ interacts with at most two unrevealed employees following the decision point; so the probability of an atypical outcome is at most $2\gamma$. Combining these observations, the focal employee's (unconditional) expected payoff $U_n$ differs from her *typical* expected payoff by at most $2\gamma \cdot (U_{\max} - U_{\min})$; and can thus be approximated (using Big-O notation) as

$$U_n = \left( U + v - V - \frac{3w}{2} - 4W - \frac{3\epsilon}{2} \right) + O(\omega, \gamma) \tag{8}$$

for some function $O(\omega, \gamma)$ where $\lim_{\gamma \to 0} \frac{O(\omega,\gamma)}{\gamma}$ is bounded and so $\lim_{\gamma \to 0} O(\omega, \gamma) = 0$.

On the other hand, suppose the focal employee violates the $R$-norm in the second action round and thus becomes $L$-revealed. Conditional on a typical outcome, the game proceeds to the second elimination round with $N_R = N_L = 2$. The focal employee survives with probability $1/2$, which occurs if the remaining unrevealed employee also becomes $L$-revealed – in which case the two $R$-revealed employees are eliminated. The focal employee's (unconditional) expected payoff is thus, via a similar calculation to (8),

$$\frac{1}{2} \left( U + v + V + 2w + 4W - 2\epsilon \right) + O(\omega, \gamma). \tag{9}$$

Note that here, as in (8), the term $O(\omega, \gamma)$ – which varies across constraints – accounts for atypical outcomes. Comparing the focal employee's payoff from following versus violating the norm, we see that following the norm is optimal if

$$U + v \geq 3V + 5w + 12W + \epsilon + O(\omega, \gamma).$$

As claimed, the incentive constraint takes the form (2).

*Example (ii):* If the focal employee is of the majority type, the relevant incentive constraints take the form (3). For example, suppose that at the start of the second action round, there is an $R$-majority with $N_R = 2$, $N_L = 1$. Let's focus on an unrevealed type-$r$ employee, and study whether it is incentive-compatible for her to follow the $R$-norm in the second action round. If the focal employee follows the $R$-norm, then (typically) the $L$-revealed employee is eliminated in the second elimination round, and nobody is subsequently eliminated (given that there is an absolute majority for types $r$ and $R$ – consisting of the two $R$-revealed employees plus the focal employee – in the third elimination round, as well as zero remaining $L$-revealed employees). Thus the focal employee's expected payoff is

$$U + v + V + 4W + 2w - \epsilon + O(\omega, \gamma).$$

On the other hand, suppose the focal employee violates the $R$-norm in the second action round by taking action $L$, and thus becomes (wrongly) $L$-revealed. Typically, in the subsequent elimination round, everyone votes their kind, except the focal employee who votes against nobody; the focal employee will survive with probability $1/2$ (which occurs if and only if the previously-unrevealed employee $L$-revealed herself); in which case nobody else is eliminated. Note that – if she survives – the focal employee's perceived type is not updated; he remains wrongly $L$-revealed at the end of the second elimination round. At the start of the third action round, $N_R = 2$ and $N_L = 3$. All employees act and vote their kind, the focal employee survives and becomes $R$-revealed, and the two $L$-revealed employees are eliminated. The focal employee's expected payoff is thus

$$\frac{1}{2} \left( U + V + v + 2w - 2\epsilon \right).$$

Combining observations, it follows that the focal employee follows the $R$-norm if and only if

$$U + V + v + 8W + 2w + O(\omega, \gamma) \geq 0;$$

as claimed, this incentive constraint takes the form (3).

Examples (i) and (ii) dealt with potential norm violations in action rounds. We also have to check that enforcing norms (by voting against revealed employees of the minority kind) is incentive-compatible in elimination rounds. The logic is similar to the action-round case above, but the there may be multiple possible ways to violate a voting norm, so we have to check each case. We illustrate with an example.

*Example (iii):* Suppose $N_R = 2$ and $N_L = 1$ at the start of the second elimination round. Focus on the case of an unrevealed type-$l$ employee and her decision in the second elimination round. The 'standard norm equilibrium' specifies that she should conform by voting like type $R$; that is, by voting against the $L$-revealed employee. If she does so, then: conditional on typical outcomes, the $L$-revealed employee is eliminated and replaced in the second elimination round. Everybody follows the $R$-norm in the third action round; the two $R$-revealed employees are eliminated with probability 1/4 in the third elimination round; and everybody acts like their kind in the private action round. The focal employee is never eliminated. The focal employee's expected payoff is thus

$$U + v - V - \frac{3w}{2} - 4W - \frac{3\epsilon}{2} + O(\omega, \gamma).$$

Now, let's enumerate the possible deviations. The focal employee may vote like type $L$ and thus become $L$-revealed. In that case, noting that the other unrevealed employee will follow the $R$-norm and vote against the initially-$L$-revealed employee, the initially-$L$-revealed employee (and nobody else) will nonetheless be eliminated in the second round. The third action round then starts with $N_R = 2$ and $N_L = 1$; the two unrevealed employees follow the norm; and the focal employee survives with probability 1/4 (in which case the two unrevealed employees reveal themselves to be kind-$L$, and the two $R$-revealed employees are eliminated in the final round). Her expected payoff is thus

$$\frac{1}{4}\left(U + v + V - 4W + 2w - \epsilon\right) - \epsilon + O(\omega, \gamma).$$

The focal employee may deviate off-path in a number of ways. She may vote against nobody; she may vote against one or both of the two $R$-revealed employees, with or without voting against the $L$-revealed employee. However, each case produces the same distribution of *typical* outcomes – because the focal employee's votes are never pivotal, whereas each deviation causes her to becomes $L$-revealed. Specifically, existing $L$-revealed employee is voted against by the two $R$-revealed employees and the remaining unrevealed employee, and thus is always voted off; whereas no other employee receives more than one vote. Thus the typical continuation outcome following each deviation is identical to that if the focal employee had instead voted like type $L$; and the focal employee's expected payoff following each deviation is identical as well. We conclude that following the norm in the second elimination round in this example is incentive-compatible if and only if

$$U + v \geq \frac{5V}{3} + 4W + \frac{8w}{3} + \frac{\epsilon}{3} + O(\omega, \gamma);$$

thus the relevant incentive constraint takes the form (2).

**Unrevealed employees (in the absence of norms)**

In rounds that start with no majority ($N_L = N_R$), the 'standard norm equilibrium' specifies that an unrevealed employee will act, and vote, with her kind. We may check that the incentive constraints at such decision points will take the form (3).

*Example (iv):* Perhaps the leading example is the first action round. There, the game always starts with $N_L = N_R = 0$: i.e., no majority. Focus on an unrevealed type-$l$ employee. Suppose, as the equilibrium specifies, she chooses course of action $L$. We enumerate the possible *typical* outcomes, and calculate the expected payoff under each outcome:

1. With probability 1/16, all four other employees become $L$-revealed (i.e., choose course of action $L$). In that case, the game ends after the first elimination round, and the focal employee's expected payoff is
$$U + V + v + 4W + 4w.$$
2. With probability 4/16, three of the other employees become $L$-revealed, and the remaining employee becomes $R$-revealed. In that case, the $R$-revealed employee is eliminated and replaced in the first elimination round; the unrevealed replacement follows the $L$-norm in the second action and elimination round; and the game ends then. The focal employee's expected payoff is
$$U + V + v + 4W + 3w + O(\omega, \gamma).$$
3. With probability 6/16, two of the other employees become $L$-revealed, and the remaining two become $R$-revealed. In that case, the $R$-revealed employees are eliminated and replaced in the first elimination round; the unrevealed replacements follow the $L$-norm in the second action and elimination round; and the game ends then. The focal employee's expected payoff is
$$U + V + v + 4W + 2w + O(\omega, \gamma).$$
4. With probability 5/16, at least three of the other employees become $R$-revealed. In that case, the focal employee is eliminated in the first round, and receives a payoff of zero.

The focal employee's unconditional expected payoff from taking course of action $L$ is thus the weighted mean of these payoffs.

On the other hand, suppose the focal employee takes action $R$ in the first round, and thus is wrongly $R$-revealed. The possible typical outcomes are:

1. With probability 1/16, all four other employees become $R$-revealed (i.e., choose course of action $L$). In that case, the focal employee will follow the $R$-norm in the first elimination round, and the game ends then. The focal employee's expected payoff is
$$U - V + v - 4W - 4w.$$
2. With probability 4/16, three of the other employees become $R$-revealed, and the remaining employee becomes $L$-revealed. In that case, the focal employee will follow the $R$-norm in subsequent rounds. The $L$-revealed employee is eliminated and replaced in the first elimination round, and the game ends after the second elimination round. The focal employee's expected payoff is
$$U - V + v - 4W - 3w + O(\omega, \gamma).$$
3. With probability 6/16, two of the other employees become $R$-revealed, and the two become $L$-revealed. In that case, the (wrongly $R$-revealed) focal employee will vote with her kind $L$ in the first elimination round, and thus become $L$-revealed; the $R$-revealed employees are

eliminated and replaced in the first elimination round; the unrevealed replacements follow the $L$-norm in the second action and elimination round; and the game ends then. The focal employee's expected payoff is

$$U + V + v + 4W + 2w + O(\omega, \gamma).$$

4. With probability 5/16, at least three of the other employees become $L$-revealed. In that case, the focal employee (being wrongly $R$-revealed) is eliminated in the first round, and receives a payoff of zero.

The focal employee's unconditional expected payoff from taking course of action $L$ is thus the weighted mean of these payoffs. Comparing her expected payoff from action $L$ versus action $R$, we find that it is optimal for her to take course of action $L$ in the first action round if and only if

$$\frac{5V}{8} + \frac{W}{2} + 2w + O(\omega, \gamma) \geq 0;$$

as claimed, this incentive constraint takes the form (3).

### Revealed employees of the minority kind

Consider a decision point where there is a "weak" $R$-majority: $N_R \geq N_L$. Our equilibrium specifies that any $L$-revealed employee will act / vote with her kind. Note that this employee's perceived type will not be updated regardless of her action or vote.

In the case that the decision point is an action round, note that the focal employee's choice of $L$ versus $R$ has no effect on her perceived type, and thus does not change the continuation equilibrium. It follows immediately that it is weakly optimal for her to act with her kind (strictly so if there is a positive probability that Part I will end in that round).

In the case that the decision point is an elimination round and there is a strict $R$-majority, $N_R > N_L$, we divide the analysis into two subcases. First, suppose $N_R \geq 3$. Then the focal employee will be eliminated with probability one, and will receive zero payoff regardless of her vote; thus it is weakly optimal for her to vote with her kind. Second, suppose $N_R = 2$ and $N_L = 1$, which can only occur at the start of the second (not the first) elimination round. Then, given that the two $R$-revealed employees will vote against the focal employee, she will not be eliminated if and only if the two unrevealed employees are strong $L$-types and thus do not vote against her. Conditioning on this possibility, it is straightforward to calculate that the incentive constraint for her to vote with her kind takes the form (3).

In the case that the decision point is a voting round and there is no majority, $N_R = N_L$, we can show, subcase-by-subcase, that the relevant incentive constraints take the form (3) as well.

### Wrongly-perceived employees of the minority kind (to follow the norm)

Consider a decision point where $N_L > N_R$, so there is an $L$-majority. If $N_L \geq N_R + 2$, our equilibrium specifies that any wrongly-perceived majority-revealed ($L$-revealed, type-$r$) employee will follow the $L$-norm. Any incentive constraint for this decision point takes the form (2). We illustrate with two examples.

*Example (v):* Consider the second action round. Suppose there is an $L$-majority with $N_L = 2$ and $N_R = 0$. Focus on a wrongly-perceived, $L$-revealed (i.e., type-$r$) employee. If she follows the

$L$-norm by taking action $L$, then in the *typical* continuation equilibrium, she (and everyone else) will also follow the $L$-norm in the second elimination round by not voting against anyone else, at which point the game ends. Her expected payoff is thus

$$U - V + v - 4W - w + O(\omega, \gamma).$$

On the other hand, if the focal employee violates the $L$-norm by taking action $R$, then in the typical continuation equilibrium, the second elimination round starts with $N_L = N_R = 1$, and everyone votes as their own kind. The focal employee survives with probability 1/2, and her expected payoff is

$$\frac{1}{2}\left(U + V + v + \frac{5W}{2} + \frac{9w}{4} - \frac{7\epsilon}{4}\right) + O(\omega, \gamma).$$

Comparing expected payoffs, it is thus optimal for her to follow the norm if

$$U + v \geq 3V + \frac{21W}{2} + \frac{17w}{4} - \frac{7\epsilon}{4} + O(\omega, \gamma);$$

that is, the incentive constraint takes the form (2), as claimed.

*Example (vi):* Again, consider the second action round, and suppose there is an absolute $L$-majority with $N_L = 4$ and $N_R = 0$. Focus on a wrongly-perceived, $L$-revealed (i.e., type-$r$) employee. If she follows the $L$-norm by taking action $L$, then in the *typical* continuation equilibrium, she (and everyone else) will also follow the $L$-norm in the second elimination round by not voting against anyone else, at which point the game ends. Her expected payoff is thus

$$U - V + v - 4W - 3w + O(\omega, \gamma).$$

On the other hand, if she violates the $L$-norm, then she is $R$-revealed, and is eliminated with probability one in the second elimination round; thus receiving zero payoff. Consequently, it is optimal to follow the $L$-norm if and only if $U + v \geq V + 4W + 3w + O(\omega, \gamma)$, which indeed takes the form (2).

What if $N_L = N_R + 1$ at a decision point? Then the 'standard norm equilibrium' specifies that (a) a wrongly-perceived majority-revealed employee will act and vote with her kind, except if (b) $N_L = 2, N_R = 1$ at the start of an elimination round – in which case a wrongly-perceived majority-revealed employee will vote as the majority kind. If (a), the incentive constraint takes the form (3); if (b), the incentive constraint takes the form (2). We illustrate (a) and (b) with examples.

*Example (vii):* Consider the second action round, and suppose that $N_L = 3$ and $N_R = 2$. Focus on a wrongly-perceived, $L$-revealed (i.e., type-$r$) employee. If (as the equilibrium specifies) she votes with her kind $R$, then in the typical continuation equilibrium: she will become $R$-revealed, and the two remaining $L$-revealed employees will be eliminated, in the second elimination round. Everyone follows the $R$-norm in the third round, and nobody is eliminated. The focal employee's expected payoff is thus

$$U + V + v + 4W + 2w - 2\epsilon + O(\omega, \gamma).$$

The focal employee may deviate from the equilibrium in multiple ways, depending on how she votes. For this example, we consider the case where she enforces the norm by voting against the two $R$-revealed employees. Then both $R$-revealed employees are eliminated; in the third action round, the focal employee (and everyone else) continue to follow the $L$-norm; in the third elimination round, she votes against the two $L$-revealed employees, who are eliminated with probability 1/4. The focal employee's expected payoff is thus

$$U - V + v - 4W - \frac{3w}{2} - \frac{5\epsilon}{2} + O(\omega, \gamma).$$

Comparing expected payoffs, it is thus optimal for her to play the equilibrium if

$$2V + 8W + \frac{7w}{2} + \frac{\epsilon}{2} + O(\omega, \gamma) \geq 0;$$

that is, the incentive constraint takes the form (3), as claimed.

*Example (viii):* Consider the second elimination round, and suppose that $N_L = 2$ and $N_R = 1$. Focus on a wrongly-perceived, $L$-revealed (i.e., type-$r$) employee. If she follows the $L$-norm by voting with her kind $L$, then in the *typical* continuation equilibrium: in the second elimination round, the sole $R$-revealed employee will be eliminated. The focal employee will follow the $L$-norm in the third round, and will be eliminated together with the other $L$-revealed employee in the third round with probability 1/8 (i.e., if all three unrevealed employees turn out to be type $r$.) The focal employee's expected payoff is thus

$$\frac{7}{8}\left(U - V + v - 4W - \frac{10w}{7}\right) - \epsilon + O(\omega, \gamma).$$

The focal employee may deviate from the equilibrium (by violating the $L$-norm) in multiple ways, depending on how she votes. However, in each case, the typical continuation equilibrium is the same: in the second elimination round, the $R$-revealed employee (and nobody else) is eliminated, and the focal employee becomes $R$-revealed. Consequently, the focal employee survives the third elimination round with probability 1/2. Her expected payoff is thus

$$\frac{1}{2}\left(U + v + V + \frac{9w}{4} + \frac{W}{4}\right) - \frac{3\epsilon}{2} + O(\omega, \gamma).$$

Comparing expected payoffs, it is thus optimal for her to follow the norm if

$$U + v \geq \frac{11V}{3} + 10W + \frac{19w}{3} - \frac{4\epsilon}{3} + O(\omega, \gamma);$$

that is, the incentive constraint takes the form (2), as claimed.

### Correctly-perceived, revealed employees of the majority kind

Consider a decision point where there is an $L$-majority: $N_L > N_R$. Our equilibrium specifies that any correctly $L$-revealed employee (i.e., type-$l$) will act / vote with her kind. In this case, the relevant incentive constraint takes the form (3).

*Example (ix):* Consider the second action round, and suppose there is an $L$-majority with $N_L = 2$ and $N_R = 1$. Focus on a correctly-perceived, $L$-revealed (i.e., type-$l$) employee. If she follows the $L$-norm by acting like kind $L$, then in the *typical* continuation equilibrium: in the second elimination round, the sole $R$-revealed employee will be eliminated. The focal employee will (with everyone else) follow the $L$-norm in the third round, and will be eliminated together with the other

$L$-revealed employee in the third round with probability 1/8 (i.e., if all then-unrevealed employees turn out to be type $r$). The focal employee's expected payoff is thus

$$\frac{7}{8}\left(U + v + V + \frac{10w}{7} + 4W\right) - \epsilon + O(\omega, \gamma).$$

If the focal employee deviates from the equilibrium (by taking action $R$) in the second action round, then she becomes (wrongly) $R$-revealed, and (typically) in the second elimination round, the remaining $L$-revealed employee (and nobody else) is eliminated. At the start of the third action round, $N_R = 2$ and $N_L = 0$; Consequently, the focal employee follows the norm in the third action round, then survives the third elimination round with probability 7/8. Her expected payoff is thus

$$\frac{7}{8}\left(U + v - V - \frac{10w}{7} - 4W\right) - \epsilon + O(\omega, \gamma).$$

Comparing expected payoffs, it is thus optimal for her to follow the norm if

$$\frac{7V}{4} + \frac{5w}{2} + 7W + O(\omega, \gamma) \geq 0;$$

that is, the incentive constraint takes the form (3), as claimed.